About one singularity of the property of disturbed differential systems, that equivalents to linear differential equations

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February 2, 2008

Abstract

This paper is devoted to the study of the family of morphisms of vector bundle, defined by the systems equivalence to linear differential equations. It is proved, that the specified families of the morphisms of vector bundle is not saturated.

1 Introduction

In what follows, I consider the families of morphisms of vector bundle [1], that defines by the linear systems of ordinary differential equations.

In [2] proved, that the families of the morphisms of vector bundles, defines by the arbitrary systems of differential equations are saturated. The property of saturation, for the first time, was defined in [3]. It turned out [4], [5], that using of this property allows to define the conditions of stability of Liapunov exponent of morphisms of vector bundles.

The aim of this paper is the study of the property of saturation conformably to the families of morphisms, that defines by the systems equivalence to linear differential equations.

2 Preliminaries

Let (E, p, B) be a vector bundle with the fibre R^n and the base B (where B is a full metric space). Fix on (E, p, B) some Riemannian metric ([6], P. 58-59).

Consider the homomorphism of group Z (group R) into group of isomorphisms of vector bundle (E, p, B). Remind, that this means following [1]: for all $t \in Z$ ($t \in R$) is given X^t - homomorphism E into E and χ^t - homomorphism B into B such that, $pX^t = \chi^t p$; for all $b \in B$ the contraction $X^t[b]$ of map X^t on fibre $p^{-1}(b)$ is linear map $p^{-1}(b) \to p^{-1}(\chi^t b)$; for all $t, s \in Z$ (correspondingly R) have a places equalities $X^{t+s} = X^t \cdot X^s$, $\chi^{t+s} = \chi^t \cdot \chi^s$.

The image of point t for this homomorphism denotes as (X^t, χ^t), instead of X^1 writes X, instead of χ^1 - χ.

Suppose, that there exists the function $a(\cdot) : B \to R^+$, that satisfies the equality $a(\chi^t b) = a(b)$ for all $b \in B$ and for all $t \in Z$ (correspondingly $t \in R$) such that, for all $t \in N$ (correspondingly $t \in R^+$) has place inequality

$$\max(\|X^t[b]\|, \|X^{-t}[b]\|) \leq e^{t \cdot a(b)} \qquad (1)$$

(the norm of a linear map of fibre into a fibre defines in the ordinary way by using norms in the fibres, induced as mentioned above fixed Riemannian metric of vector bundle (E, p, B)).

Assign for all $m \in N$

$$(X(m), \chi(m)) \overset{\text{def}}{=} (X^m, \chi^m).$$

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Obtained in this way the family of morphisms (X^m, χ^m) $m \in N$ of vector bundle (E, p, B), that satisfies the following condition: there exists the function $a(\cdot) : B \to R^+$ such that, for all $b \in B$, $m \in N$ has place inequality

$$\max(\|X(m, b)\|, \|[X(m, b)]^{-1}\|) \leq e^{m \cdot a(b)}, \tag{2}$$

where as $X(m, b)$ denotes the contraction of map $X(m)$ on fibre $p^{-1}(b)$; thus, if $X(m) = X^m$, then $X(m, b) = X^m[b]$. Moreover, such defined family of morphisms $(X(m), \chi(m))$ $(m \in N)$ satisfies the conditions a) - c):

a) (X, χ) - isomorphism of vector bundle (E, p, B);

b) for all $m \in N$ have places equalities

$$X(m) = X^m, \ \chi(m) = \chi^m;$$

c) there exists the function $a(\cdot) : B \to R^+$ such that, $a(\chi^m b) = a(b)$ for all $b \in B$, $m \in Z$, and so that, for all $b \in B$ has place inequality

$$\max(\|X[b]\|, \|[X[b]]^{-1}\|) \leq e^{a(b)}. \tag{3}$$

The verification of all this claims is trivial.

Remind the definition of saturated families of morphisms, given in [3]:

Definition *The family of morphisms $(X(m), \chi(m)) : (E, p, B) \to (E, p, B)$ $(m \in N)$, that satisfies the conditions a) - c), is said to be saturated, if for any point $b \in B$ such that $\chi^m b \neq b$ for $\forall m \neq 0$, for $\forall \varepsilon > 0$, for any basis $\{\xi_1, \ldots, \xi_n\}$ of vector space $p^{-1}(b)$ and for all neighbourhoods $U(\xi_i)$ of points ξ_i $(i \in \{1, \ldots, n\})$ (in the space E) there exists $\delta > 0$ such that for $\forall \bar{t} \in N$ and for all nonsingular linear operators*

$$Y_m : p^{-1}(\chi^{m-1}b) \to p^{-1}(\chi^m b)$$

$(m \in \{1, \ldots, \bar{t}\})$, that satisfy for $\forall m \in \{1, \ldots, \bar{t}\}$ the inequality

$$\|Y_m(X[\chi^{m-1}b])^{-1} - E\| + \|X[\chi^{m-1}b]Y_m^{-1} - E\| < \delta \tag{4}$$

there exists the point $b' \in B$ such that

$$d_B(b', b) < \varepsilon, \tag{5}$$

and for $\forall m \in \{0, \ldots, \bar{t}\}$ there exist isomorphisms of fibres (as Cartesian spaces)

$$\psi_m : p^{-1}(\chi^m b') \to p^{-1}(\chi^m b),$$

in which connection fulfiled following conditions:

i) $\psi_0^{-1}\xi_i \in U(\xi_i)$ for $\forall i \in \{1, \ldots, n\}$;

ii) for $\forall m \in \{1, \ldots, \bar{t}\}$ diagram

$$
\begin{CD}
p^{-1}(\chi^{m-1}b') @>{X[\chi^{m-1}b']}>> p^{-1}(\chi^m b') \\
@V{\psi_{m-1}}VV @VV{\psi_m}V \\
p^{-1}(\chi^{m-1}b) @>{Y_m}>> p^{-1}(\chi^m b)
\end{CD}
\tag{6}
$$

is commutative.

Consider the family of morphisms defined in [7].

Let the space of all linear systems of differential equations

$$\dot{x} = A(t) \cdot x, \ x \in R^n, \tag{7}$$

such that $A(\cdot) : R \to \operatorname{Hom}(R^n, R^n)$ - continuous map, for which $\sup_{t \in R} \|A(t)\| < +\infty$, endow with a structure of the metric space by define distance by formula

$$d(A_1, A_2) = \sup \|A_1(t) - A_2(t)\|.$$

(here point $\dot{x} = A_i(t) \cdot x$ of space denotes by A_i). Obtained metric space M_n is full.

Let

$$B \stackrel{\text{def}}{=} M_n, \ E \stackrel{\text{def}}{=} B \times R^n, \ p \stackrel{\text{def}}{=} pr_1, \tag{8}$$

where pr_1 - projection of production $B \times R^n$ on the first efficient.

Thus, the trivial vector bundle (E, p, B) is defined.

Let for all $t \in R$

$$X^t(A, x) = (\chi(t)A, \mathfrak{X}(t, 0, A)x), \tag{9}$$

$$\chi^t A(\cdot) = A(t + (\cdot)), \tag{10}$$

where $A \in B$, $x \in R^n$, $\mathfrak{X}(\Theta, \tau, A)$ - the Cauchy matrix of a system $\dot{x} = A(t) \cdot x$.

For all $t \in R$ have places formulas:

$$X^t \cdot X^{-t} = X^0 = 1_E, \ X^{-t} \cdot X^t = X^0 = 1_E,$$

$$\chi^t \cdot \chi^{-t} = \chi^0 = 1_B, \ \chi^{-t} \cdot \chi^t = \chi^0 = 1_B,$$

i.e. (X^{-t}, χ^{-t}) - morphism of vector bundle (E, p, B), inverse for (X^t, χ^t), therefor, (X^t, χ^t) - isomorphism of vector bundle (E, p, B).

Thus, the homomorphism of group R into group of isomorphisms of vector bundle (E, p, B) is defined. The image of point $t \in R$ for this homomorphism is (X^t, χ^t).

The function $a(\cdot) : B \to R^+$ defines by formula

$$a(A) \stackrel{\text{def}}{=} \sup_{t \in R} \|A(t)\|.$$

We have

$$a(\chi^t A) = a(A) = \sup_{t \in R} \|A(t)\|.$$

The correctness for all $b \in B$, $t \in R^+$ of inequality (1) follows from (9), in view of well-known inequality

$$\|\mathfrak{X}(t, 0, A)\| \le e^{|\int_0^t \|A(s)ds\||} \le e^{|t \cdot a(A)|},$$

to which Cauchy matrix of system $\dot{x} = A(t) \cdot x$ satisfies for all $A \in B$, $t \in R$.

Let

$$(X(m), \chi(m)) \stackrel{\text{def}}{=} (X^m, \chi^m).$$

We obtain the family of morphisms

$$(X(m), \chi(m)) : (E, p, B) \to (E, p, B),$$

$(m \in N)$, that satisfies the conditions a) - c).

In [2] proved, that the family of morphisms (8) is saturated.

3 Auxiliary results

Consider the differential equation

$$\ddot{y} = a(t) \cdot y, \ y \in R, \tag{11}$$

where $a(\cdot) : R \to R$ - continues map with $\sup_{t \in R} |a(t)| < \infty$.

The equation (11) equivalents to the system of differential equations

$$\dot{x} = A(t)x, \ x \in R^2, \ A(t) = \begin{pmatrix} 0 & 1 \\ a(t) & 0 \end{pmatrix}. \tag{12}$$

Consider also the differential equations

$$\ddot{y} = [a(t) + b_\varepsilon(t)] \cdot y, \ y \in R, \tag{13}$$

where $b_\varepsilon(\cdot) : R \to R$ - continues map with $\sup\limits_{t \in R} |b_\varepsilon(t)| < \varepsilon$ $(\varepsilon > 0)$.

The equation (13) equivalents to system of differential equations

$$\dot{x} = [A(t) + B_\varepsilon(t)] \cdot x, \ x \in R^2, \tag{14}$$

where matrix $B_\varepsilon(t)$ has representation and estimation

$$B_\varepsilon(t) = \begin{pmatrix} 0 & 0 \\ b_\varepsilon(t) & 0 \end{pmatrix}, \ \sup\limits_{t \in R} \|B_\varepsilon(t)\| < \varepsilon. \tag{15}$$

Lemma 1. *If the differential equation (11) is given and $\mathfrak{X}(\Theta, \tau, A)$ - the Cauchy matrix of a equivalent system (12), then for $\forall \varepsilon > 0$ there exists $\delta > 0$ such that for $\forall \overline{t} \in N$ there always exists nonsingular linear operators $W_m : R^2 \to R^2$ $(m \in \{1, \dots, \overline{t}\})$ that satisfy for $\forall m \in \{1, \dots, \overline{t}\}$ the inequality*

$$\|W_m[\mathfrak{X}(m, m-1, A)]^{-1} - E\| + \|\mathfrak{X}(m, m-1, A)W_m^{-1} - E\| < \delta \tag{16}$$

for which there no exists continues map

$$A_\varepsilon(\cdot) : [0, \overline{t}] \to \mathrm{Hom}(R^2, R^2)$$

that satisfies the condition:
1) $\sup\limits_{t \in [0, \overline{t}]} \|A_\varepsilon(t) - A(t)\| < \varepsilon$;
2) $\mathfrak{X}(m, m-1, A_\varepsilon) = W_m$ *for* $\forall m \in \{1, \dots, \overline{t}\}$,
where $\mathfrak{X}(m, m-1, A_\varepsilon)$ - *Cauchy matrix of system (14), that equivalents to equation (13).*

Proof. We argue by contradiction. Then for $\forall \varepsilon > 0$ there exists $\delta > 0$ such that for $\forall \overline{t} \in N$ and for all nonsingular linear operators $W_m : R^2 \to R^2$ $(m \in \{1, \dots, \overline{t}\})$, that satisfy the inequality (16), there exists continues map $A_\varepsilon(\cdot) : [0, \overline{t}] \to \mathrm{Hom}(R^2, R^2)$, that satisfies the conditions 1), 2).

Let $\mathfrak{X}(t, s, A_\varepsilon)$ be Cauchy matrix of system (14). Then for $\forall t, s \in R$ has place equality:

$$\dot{\mathfrak{X}}(t, s, A_\varepsilon) = A_\varepsilon(t) \cdot \mathfrak{X}(t, s, A_\varepsilon) = [A(t) + B_\varepsilon(t)] \cdot \mathfrak{X}(t, s, A_\varepsilon).$$

If we multiply last equality from the right by $\mathfrak{X}^{-1}(t, s, A_\varepsilon)$, then obtain equality

$$B_\varepsilon(t) = \dot{\mathfrak{X}}(t, s, A_\varepsilon) \cdot \mathfrak{X}^{-1}(t, s, A_\varepsilon) - A(t), \tag{17}$$

correctness for $\forall t, s \in R$.

Consider the equation

$$\ddot{y} = \frac{2t^2 - 1}{(1 + t^2)^2} \cdot y. \tag{18}$$

For the equation (18) the equivalence system is

$$\dot{x} = A(t) \cdot x, \ A(t) = \begin{pmatrix} 0 & 1 \\ \frac{2t^2-1}{(1+t^2)^2} & 0 \end{pmatrix}, \ x \in R^2. \tag{19}$$

The Cauchy matrix of system (19) has a representation:

$$\mathfrak{X}(t, s, A) = \frac{1}{3\sqrt{(1+t^2)(1+s^2)}} \cdot \begin{pmatrix} \frac{f_1(t,s)}{1+s^2} & -3s - s^3 + 3t + t^3 \\ \frac{f_2(t,s)}{(1+t^2)(1+s^2)} & \frac{3+3ts+ts^3+3t^2+2t^4}{1+t^2} \end{pmatrix}, \tag{20}$$

where

$$f_1(t, s) = 3 + 3s^2 + 2s^4 + 3ts + t^3 s,$$

$$f_2(t, s) = -3t - 3ts^2 - 2ts^4 + 3s + 3st^2 + 2st^4.$$

Assign for $\forall m \in \{1, \ldots, \bar{t}\}$ $(\bar{t} \in N)$

$$W_m = \mathfrak{X}(m, m-1, A) + \begin{pmatrix} 0 & 0 \\ 0 & \frac{r_m}{3\sqrt{(1+m^2)^3(1+(m-1)^2)}} \end{pmatrix}, \tag{21}$$

where $r_m > 0$ - some real numbers, that depend on δ and m.

We have

$$W_m \cdot [\mathfrak{X}(m, m-1, A)]^{-1} = \frac{r_m}{g_1(m, m-1)} \cdot \begin{pmatrix} 0 & 0 \\ -f_2(m, m-1) & f_1(m, m-1) \end{pmatrix}, \tag{22}$$

$$\mathfrak{X}(m, m-1, A) \cdot W_m^{-1} = \frac{r_m}{g_2(m, m-1)} \cdot \begin{pmatrix} 0 & 0 \\ f_2(m, m-1) & f_1(m, m-1) \end{pmatrix}, \tag{23}$$

where

$$g_1(t, s) = 9 \cdot (1 + t^2)^3 \cdot (1 + s^2)^2,$$

$$g_2(t, s) = (1 + t^2) \cdot (g_3(t, s) + f_1(t, s) \cdot r),$$

$$g_3(t, s) = 9(1 + 2t^2 + t^4 + 4s^2t^2 + 2s^2t^4 + 2s^4t^2 + s^4t^4 + s^2 + s^4).$$

If now assign, that the r_m satisfies the inequality

$$r_m < \frac{\delta}{2 \max\{|f_2(m, m-1)|, |f_1(m, m-1)|\}} \times$$

$$\times \max\{g_1(m, m-1), \ (1 + m^2) \cdot (g_3(m, m-1) + |f_1(m, m-1)| \cdot r_m)\},$$

then obtain, that the operators W_m for $\forall m \in \{1, \ldots, \bar{t}\}$ satisfy the inequality (16).

Then in suppose, for all $m \in \{1, \ldots, \bar{t}\}$

$$\mathfrak{X}(m, m-1, A_\varepsilon) = W_m$$

and in view of (17), (21) and (20) we have

$$B_\varepsilon(m) = \dot{W}_m \cdot W_m^{-1} - A(m) =$$

$$= \frac{r_m}{g_2(m, m-1)} \cdot \begin{pmatrix} f_2(m, m-1) & -f_1(m, m-1) \cdot (1 + m^2) \\ \frac{3f_2(m, m-1) \cdot m}{1+m^2} & 3f_1(m, m-1) \cdot m \end{pmatrix}. \tag{24}$$

In view of (13), the matrix $B_\varepsilon(t)$ must has a form (15). But the right side of (24) don't satisfies the (15). This contradiction proves a lemma.

Consider the family of morphisms $\mathfrak{G}$

$$(X(m), \chi(m)): \quad (E, p, B) \to (E, p, B), \tag{25}$$

$(m \in N)$, of vector bundle (E, p, B), in which connection

$$B = M_2, \quad E = B \times R^2, \quad p = pr_1, \tag{26}$$

$$X^t(A, x) = (\chi^t A, \mathfrak{X}(t, 0, A) \cdot x), \tag{27}$$

$$\chi^t A(\cdot) = A(t + (\cdot)), \tag{28}$$

where $A \in B$, $x \in R^2$, $\mathfrak{X}(\Theta, \tau, A)$ - Cauchy matrix of the system (12), that equivalents to equation (11).

It is easy to make sure that the family of morphisms (25), (27), (28) satisfies the conditions a) - c).

4 Two-dimensional case

The aim of this section is to prove the following result.

Lemma 2. *The family of morphisms $\mathfrak{G}$ of vector bundle (26) is not saturated.*

Proof. We argue by contradiction. Suppose, that the family of morphisms (25), (27), (28) of vector bundle (26) is saturated.

Then in view of definition, for any point $b \in B$ such that $\chi^m b \neq b$ for $\forall m \neq 0$, for $\forall \varepsilon > 0$ for any basis $\{\xi_1, \xi_2\}$ of vector space $p^{-1}(b)$ and for all neighbourhoods $U(\xi_i)$ of points ξ_i ($i = \overline{1,2}$)(in space E) there exists $\delta > 0$ such that for $\forall t \in N$ and for all nonsingular linear operators $Y_m : p^{-1}(\chi^{m-1}b) \to p^{-1}(\chi^m b)$ ($m \in \{1, \ldots, \overline{t}\}$), that satisfy for $\forall m \in \{1, \ldots, \overline{t}\}$ the inequality (4) there exists the point $b' \in B$ such that $d_B(b, b') < \overline{\varepsilon}$ and for $\forall m \in \{0, \ldots, \overline{t}\}$ there exist isomorphisms of fibres (as a Cartesian spaces) $\psi_m : p^{-1}(\chi^m b') \to p^{-1}(\chi^m b)$, in which connection, have places conditions i) - ii).

Let $\overline{\varepsilon} = \varepsilon$ and select $\delta > 0$ and $\overline{t}$ such that has place the condition of lemma 1.

Fix in the space B any point $b = A$, and then select in the space $p^{-1}(b)$ any basis $\{\xi_1, \xi_2\}$.

Consider the map $Y_m : p^{-1}(\chi^{m-1}b) \to p^{-1}(\chi^m b)$

$$Y_m(A, \xi) = (\chi^m A, W_m \xi), \tag{29}$$

where $A \in B$, $\xi \in R^2$, $W_m = W(m, m-1)$ ($m \in \{1, \ldots, \overline{t}\}$) - any nonsingular operators from lemma 1.

It is easy to make sure, that in such case the operator Y_m satisfies the inequality (4).

Then, in view of supposition, there exists the point $b' \in B$ such that $d_B(b, b') < \overline{\varepsilon}$ and for $\forall m \in \{0, \ldots, \overline{t}\}$ there exist isomorphisms of fibres $\psi_m : p^{-1}(\chi^m b') \to p^{-1}(\chi^m b)$, in which connection have places conditions i) - ii).

Let $b' = A_\varepsilon$, where $A_\varepsilon = A_\varepsilon(t)$ - operator from lemma 1.

Since $\overline{\varepsilon} = \varepsilon$, then condition $d_B(b, b') < \overline{\varepsilon}$ has place.

The condition ii) supposes, that diagram (6) is commutative, whence, taking into consideration (27), (29), obtain:

$$
\begin{array}{ccc}
p^{-1}(\chi^{m-1}b') & \xrightarrow{X[\chi^{m-1}b']} & p^{-1}(\chi^m b') \\
\downarrow{\scriptstyle p_{2,\chi^{m-1}b'}} & & \downarrow{\scriptstyle p_{2,\chi^m b'}} \\
R^2 & \xrightarrow{\mathfrak{X}(m,m-1,A_\varepsilon)} & R^2
\end{array}
\tag{30}
$$

$$
\begin{array}{ccc}
p^{-1}(\chi^{m-1}b) & \xrightarrow{Y_m} & p^{-1}(\chi^m b) \\
\downarrow{\scriptstyle p_{2,\chi^{m-1}b}} & & \downarrow{\scriptstyle p_{2,\chi^m b}} \\
R^2 & \xrightarrow{W_m} & R^2
\end{array}
\tag{31}
$$

where $p_{2,\overline{b}}$ - the contraction on fibre $p^{-1}(\overline{b})$ of map pr_2 (pr_2 - the projection of product by second efficient).

From the commutative of diagrams (6), (30), (31) follows commutative of diagram

$$
\begin{array}{ccc}
R^2 & \xrightarrow{\mathfrak{X}(m,m-1,A_\varepsilon)} & R^2 \\
{\scriptstyle p_{2,\chi^{m-1}b}\cdot\psi_{m-1}\cdot(p_{2,\chi^{m-1}b'})^{-1}}\downarrow & & \downarrow{\scriptstyle p_{2,\chi^m b}\cdot\psi_m\cdot(p_{2,\chi^m b'})^{-1}} \\
R^2 & \xrightarrow{W_m} & R^2
\end{array}
\tag{32}
$$

From where follows a claim: for $\forall \varepsilon > 0$ $\exists \delta > 0$ such that for $\forall t \in N$ there always exist nonsingular linear operators $W_m : R^2 \to R^2$ ($m \in \{1, \ldots, \overline{t}\}$), that satisfy for $\forall m \in \{1, \ldots, \overline{t}\}$ the inequality (16), for which there no exists the continues map $A_\varepsilon(\cdot) : [0, \overline{t}] \to \text{Hom}(R^2, R^2)$, that satisfies the conditions 1) - 2). Since obtained claim is contrary to lemma 1, then lemma is proved.

5 n-dimensional case

Now consider the linear differential equations

$$y^{(n)} + a_1(t)y^{(n-1)} + \ldots + a_n(t)y = 0, \tag{33}$$

where $\sup\limits_{t} |a_i(t)| < \infty$, $(i = \overline{1,n})$ and equivalence to them systems of differential equations

$$\dot{x} = A(t) \cdot x, \quad x \in R^n. \tag{34}$$

On the set of systems (34) defines the metric. The obtained metric space denotes as M_n.

The family of morphisms

$$(X(m), \chi(m)) : (E, p, B) \to (E, p, B),$$

$(m \in N)$, of vector bundle (E, p, B), where

$$B = M_n, \quad E = B \times R^n, \quad p = pr_1, \tag{35}$$

$$X^t(A, x) = (\chi^t A, \mathfrak{X}(t, 0, A) \cdot x),$$

$$\chi^t A(\cdot) = A(t + (\cdot)),$$

denotes as $\mathfrak{S}$ (here $A \in B$, $x \in R^n$, $\mathfrak{X}(\Theta, \tau, A)$ - Cauchy matrix of system (34), that equivalents to equation (33)).

Theorem 1. *If the differential equation (33) is given and $\mathfrak{X}(\Theta, \tau, A)$ - a Cauchy matrix of equivalence system (34), then for $\forall \varepsilon > 0$ there exists $\delta > 0$ such that for $\forall \bar{t} \in N$ there always exist nonsingular linear operators $W_m : R^n \to R^n$ $(m \in \{1, \ldots, \bar{t}\})$ that satisfy for $\forall m \in \{1, \ldots, \bar{t}\}$ the inequality (16), for which there no exists continues map*

$$A_\varepsilon(\cdot) : [0, \bar{t}] \to \mathrm{Hom}(R^n, R^n)$$

that satisfies the conditions:

I) $\sup\limits_{t \in [0, \bar{t}]} \|A_\varepsilon(t) - A(t)\| < \varepsilon$;

II) $\mathfrak{X}(m, m - 1, A_\varepsilon) = W_m$ for $\forall m \in \{1, \ldots, \bar{t}\}$,

where $\mathfrak{X}(m, m - 1, A_\varepsilon)$ - Cauchy matrix of system (34) that equivalents to equation (33).

Proof. Consider the equation

$$y^{(n)} = a(t)y^{(n-2)}, \quad y \in R. \tag{36}$$

The system, that equivalents to (36), have a representation:

$$\dot{x} = A(t) \cdot x, \quad x \in R^n,$$

where

$$A(t) = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 0 & \ldots & a(t) & 0 \end{pmatrix}.$$

Suppose, that has place the claim, that contraries to claim of theorem. It means, that for $\forall \bar{t} \in N$ for all linear operators W_m, that satisfy for $\forall m \in \{1, \ldots, \bar{t}\}$ the inequality (16), there always exists a matrix $B_\varepsilon(t) = A_\varepsilon(t) - A(t)$ with representation

$$B_\varepsilon(t) = \begin{pmatrix} 0 & \ldots & 0 & 0 \\ 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & b_\varepsilon(t) & 0 \end{pmatrix}, \quad \sup\limits_{t} |b_\varepsilon(t)| < \varepsilon,$$

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such that conditions I), II) is satisfied.

On the other hand, if in the equation (36) we make change $y^{(n-2)} = z$, then we obtain already considered second order equation

$$\ddot{z} = a(t) \cdot z, \quad z \in R^2.$$

For this equation, according to lemma 1, for $\forall \bar{t} \in N$ there always exist the linear operators W_m, that satisfy for $\forall m \in \{1, \ldots, \bar{t}\}$ the inequality (16), for which there no exists the matrix $B_\varepsilon(t)$, that have representation (15), such that the conditions 1), 2) is satisfied. This contradiction proves theorem.

Theorem 2. *The family of morphisms $\mathfrak{S}$ of vector bundle (35) is not saturated.*

The proof of theorem is similarly to proof of lemma 2 and bases on theorem 1 validity.

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